PHYSICAL REVIEW D, VOLUME 62, 057901

No CPT violation from a tilted brane in neutral meson-antimeson systems

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(Received 18 April 2000; published 11 August 2000)

A tilted brane in theories with large compact extra dimensions leads to spontaneous symmetry breaking of the Lorentz and rotational invariance in four dimensions, as shown by Dvali and Shifman. In this Brief Report, we point out that the mentioned Lorentz symmetry breaking, although leading to CPT-violating interaction terms, cannot lead to CPT violation in the experimentally interesting K-$\bar{K}$ and analogous systems.

PACS number(s): 11.25.Mj, 11.30.Er, 13.20.Gd

Recently, the authors of [1] pointed out that in theories with large compact extra dimensions [2] there exist solutions for the 3-brane with a constant gradient energy—a tilted brane. Further, they argued that such brane solutions, via the change of the metric on the brane, lead in four dimensions to interaction terms which violate Lorentz and rotational invariance.

In this Brief Report, we show that such interaction terms lead in principle to terms which violate CPT invariance. Subsequently, we argue that this framework leads nonetheless to no CPT violation effects in the K-$\bar{K}$ and analogous neutral meson-antimeson systems.

The tilted brane can be described by a low energy effective theory. In this theory, the graviphoton $\mathcal{A}_\mu(x)$ obtains a mass $m_{\mathcal{A}}\sim 1\ \mathrm{mm}^{-1}$ by eating up the corresponding Goldstone mode. After integrating out the graviphoton, the effective theory for the other pseudo Nambu-Goldstone boson(s) (pNGB) χ describing the dynamics of the brane is represented by the four-dimensional Lagrangian density

$$\mathcal{L}_{\mathrm{brane}}=g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi, \tag{1}$$

where $g_{\mu\nu}$ is the induced metric on the brane. There are also fermionic terms, but they are not relevant unless we include supersymmetry. The tilted brane solution to the equation of motion $\partial^2\chi=0$ is

$$\chi=\sqrt{T}\alpha x^j, \tag{2}$$

where $\hat{x}^j$ is the direction along which the tilting occurs, T is the brane tension, i.e., the energy per 3-space unit volume, and α is the angle of the tilting. The following relations hold: $T\sim M_{\mathrm{P_f}}^4$, where $M_{\mathrm{P_f}}\gtrsim 1$ TeV ($\sim 10^{16}$ mm) is the fundamental scale of gravity in the framework; $\alpha\sim RH$, where R is the typical size of the extra dimension(s) ($R\lesssim 1$ mm) and H^{-1} is the Hubble size ($\sim 10^{28}$ mm).

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The tiny tilting angle α causes the brane to be ‘‘stretched’’ by the factor of $1+\alpha^2/2$. On the other hand, $\alpha=T^{-1/2}\partial\chi/\partial x^j$ by Eq. (2). This implies that the induced metric $g_{\mu\nu}$ and the flat (untilted) metric $g_{\mu\nu}^{(0)}$ are related

$$g_{\mu\nu}=g_{\mu\nu}^{(0)}+\frac{1}{2}T^{-1}\partial_\mu\chi\partial_\nu\chi. \tag{3}$$

An analogous relation for the basis 4-vectors e_a on the tilted brane

$$e_a^\mu=g_a^{(0)\mu}+\frac{1}{4}T^{-1}\partial^\mu\chi\partial_a\chi \tag{4}$$

follows from Eq. (3) due to $e_a\cdot e_b=g_{ab}$. The kinetic term for the fermionic fields on the tilted brane involves $\not{\partial}_{\mathrm{tilted}}=e_a^\mu\gamma^a\partial_\mu$. Therefore, when using Eq. (4), the kinetic energy term in the tilted brane background can be rewritten as

$$\begin{aligned}\mathcal{L}_{\mathrm{kin.}}&=(\bar{\psi}\not{\partial}\psi)_{\mathrm{tilted}}\\&=\bar{\psi}\gamma^\mu\partial_\mu\psi+\frac{1}{4}T^{-1}(\partial^\mu\chi\partial_\nu\chi)(\bar{\psi}\gamma^\nu\partial_\mu\psi),\end{aligned} \tag{5}$$

where all the derivatives are in the flat metric. If we now expand around the tilted brane solution (2), i.e., $\chi=\sqrt{T}\alpha x^j+\delta\chi$, we obtain from the last term of Eq. (5) interaction terms which break the Lorentz and rotational invariance

$$\begin{aligned}\delta\mathcal{L}=&\frac{1}{4}T^{-1/2}\alpha\partial_\nu(\delta\chi)[\bar{\psi}(\gamma^\nu\partial^j+\gamma^j\partial^\nu)\psi]\\&+\frac{1}{4}\alpha^2\bar{\psi}\gamma^\nu\partial^\mu\psi|_{\nu=\mu=j}.\end{aligned} \tag{6}$$

The first term ($\propto\alpha$), in addition, violates CPT, because $\partial_\nu(\delta\chi)$ is odd and the term in $[\cdots]$ is even under CPT. The violation of the Lorentz symmetry and of CPT usually go hand in hand [3]. The last term in Eq. (6) is CPT even.

The question immediately arising at this point is whether the above CPT-violating interaction can lead to CPT-violating effects in the K-$\bar{K}$ or analogous neutral meson-antimeson systems. CPT violation phenomena in the latter systems are in principle experimentally detectable and were discussed in Ref. [4]. The current experimental bound on CPT violation in the K-$\overline{K}$ system is already very stringent [5]

$$\frac{m_K - m_{\overline{K}}}{m_K} < 2 \times 10^{-18}. \tag{7}$$

As argued in Ref. [4], in string theory it is possible to have a breaking of CPT that would lead to nonzero energy shifts in the the quark inverse propagators. If the 4-dimensional effective theory arises from a string theory compactified at the Planck scale $M_{\rm Pl} \sim 10^{19}$ GeV, this may lead, among other things, to 4-dimensional effective interactions of the form [4]

$$\mathcal{L}_I \sim \mathcal{T} \overline{\psi} \Gamma \psi, \tag{8}$$

where the field $\mathcal{T}$ is a CPT-odd 4-dimensional Lorentz tensor, and Γ denotes a gamma-matrix structure. If tensor $\mathcal{T}$ acquires a nonzero vacuum expectation value $\langle \mathcal{T} \rangle$, the interaction (8) generates a quadratic purely fermionic term and thus leads to a tree level nonzero contribution ΔK to the fermion inverse propagator. An effective ''stringy'' interaction that would lead to nonzero energy shifts in the quark inverse propagators is

$$\delta \mathcal{L} \propto \langle \mathcal{T}_{000} \rangle_0 \overline{\psi} \gamma^0 \psi, \tag{9}$$

where $\langle \mathcal{T} \rangle \propto 1/M_{\rm Pl}$. The effective interaction (9) is CPT-violating since $\overline{\psi}\gamma^0\psi$ is CPT-odd. This term directly contributes to shifting the energies of the quark inverse propagators—for quark q and antiquark $\overline{q}$ with opposite signs. Therefore, such operator can contribute to nonzero mass difference $m_K - m_{\overline{K}}$.

Now, let us check whether such a shift in the quark inverse propagators can arise from the CPT-violating term in Eq. (6). That term still has one pNGB-field χ (of the tilted brane) present there beside the quadratic quark field structure—i.e., in contrast to Eq. (9) it is not purely quadratic quark field term. Therefore, it cannot contribute at the tree level to energy or momentum shifts in the quark inverse propagators. It can contribute to the quark inverse propagators only through contraction of the χ fields, which corresponds to the one-loop contribution mediated by the χ field. However, this can only lead to CPT-conserving phenomena because the CPT-odd term of Eq. (6) is then applied twice (in general: an even number of times). Therefore, the CPT-odd term (6) in the tilted brane scenario cannot lead to CPT-violating effects in the neutral meson-antimeson systems, i.e., it cannot induce nonzero mass difference $m_K - m_{\overline{K}}$. The terms in Eq. (6) may still lead to other CPT-violating phenomena in other systems [6].

In summary, we showed that the metric-induced terms which violate Lorentz symmetry in the tilted brane scenario by Dvali and Shifman, while being CPT-odd, lead to no CPT-violating effects in the neutral meson-antimeson systems.

The work of G.C. was supported by the KOSEF Brainpool Program. The work of C.S.K. was supported in part by Grant No. 1999-2-111-002-5 from the Interdisciplinary Research Program of the KOSEF, in part by the BSRI Program, Ministry of Education, Project No. 99-015-DI0032, in part by Sughak program of Korean Reasearch Foundation, Project No. 1997-011-D00015, and in part by Brain Korea 21 Project. The work of K.L. was supported by the KOSEF Interdisciplinary Research Program.

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